

Neutral mesons and disconnected diagrams in Twisted Mass QCD

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We evaluate properties of neutral mesons in $N_f = 2$ dynamical simulations of TMQCD at maximal twist. The pion is explored - establishing the size of the isospin splitting (an order a^2 effect). We investigate the η' (the $N_f = 2$ flavour singlet pseudoscalar meson) and neutral ρ and scalar mesons. We show that disconnected diagrams can be evaluated very efficiently in TMQCD using variance reduction methods.

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1. Introduction

Here we discuss TMQCD at maximal twist with $N_f = 2$ degenerate sea quarks using configurations from ETMC [1–3]. In particular we focus on neutral mesons, which have some unusual properties in TMQCD. Thus we evaluate the disconnected contributions which are needed for a study of neutral mesons [4]. We present a new method which, for twisted mass, allows many disconnected contributions to be evaluated very efficiently.

We then present results for pions (exploring the charge splitting between π^0 and π^+). We also present results for flavour singlet pseudoscalar mesons (the η_2 meson), for vector mesons (charge splitting and decays), and for flavour-singlet scalar mesons.

2. Disconnected diagrams - variance reduction

TMQCD has a degenerate pair of u, d quarks with fermion matrix: $M_{u,d} = M_W \pm i\mu\gamma_5$ where M_W is the Wilson-Dirac matrix. Hence $1/M_u - 1/M_d = -2i\mu(1/M_d)\gamma_5(1/M_u)$.

Consider the disconnected contribution $\sum X(1/M_u - 1/M_d)$ where X is some γ -matrix and/or colour-matrix and the sum is over space. The conventional method involves solving $\phi_r = (1/M_u)\xi_r$ with stochastic volume sources ξ_r , then $\sum X/M_u = \sum \langle \xi^* X \phi \rangle_r$ where the average is over noise samples (labelled r). However, the case mentioned above can be evaluated efficiently using the ‘one-end-trick’ [5, 6]. Then the required disconnected loop is given by

$$\sum X(1/M_u - 1/M_d) = -2i\mu \sum \langle \phi^* X \gamma_5 \phi \rangle_r$$

This has signal/noise $V/\sqrt{V^2} = 1$ which is much more favourable than the conventional method with signal/noise $1/\sqrt{V}$ (here $V = L^3 T$).

For example, at $\beta = 3.9$, $\mu = 0.004$, $L = 24$, (where $M(\pi) \approx 300$ MeV, $a = 0.086$ fm, $La = 2.1$ fm) taking $X = i$ which is appropriate for the η_2 correlator, we evaluate the momentum-zero loop at a time-slice. We find a standard deviation of $\sigma = 18$ from inherent gauge-time variation whereas the stochastic noise is $\sigma = 87$ from 24 samples of volume source (conventional method) but only $\sigma = 7.5$ from above method (with 12 samples). So 12 inversions give the disconnected correlator from all t to all t' with no significant increase in errors from the stochastic evaluation.

For cases where this method cannot be used, eg. π^0 with $\bar{\psi}\gamma_5\tau_3\psi \rightarrow \bar{\chi}I\chi$ at maximal twist, we use hopping parameter variance reduction [7] instead.

3. Pion order(a^2) effects

For ETMC data with $M(\pi^+) \approx 300$ MeV, $a = 0.086$ fm and $La = 2.1$ fm, we show the ratios of correlators in fig. 1. The disconnected pieces are seen to be relatively large - and reduce the charge splitting as found previously [4]. The π^0 is lighter than the π^+ , unlike a previous preliminary study of dynamical fermions [8]. From a 4×4 fit to these correlations, we obtain $\Delta ma = 0.027(7)$. Since we expect $r_0^2(m(\pi^0)^2 - m(\pi^+)^2) = c(a/r_0)^2$, we compare this expression with results from several lattice data sets [2] in fig. 2.

We see that, as expected, the flavour splitting decreases as a^2 . The sign and behaviour are consistent with Chiral PT and the nature of the phase transition [1] where $m(\pi^0) = 0$. We can use

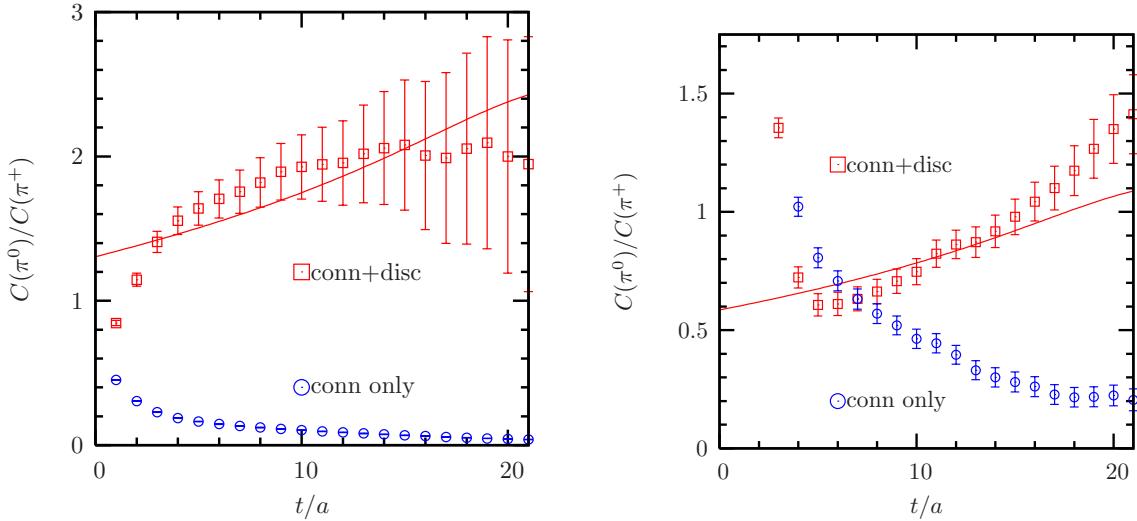


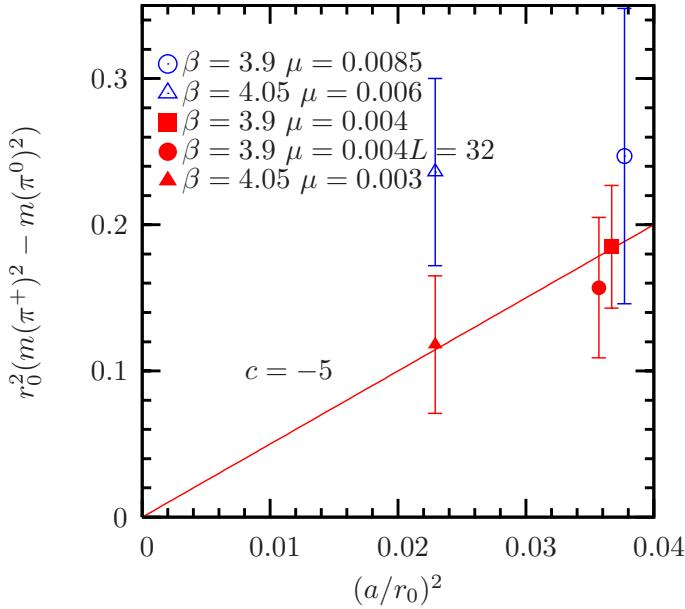
Figure 1: Ratio of correlators between neutral and charged pions. The left plot is for local operators $\bar{\chi}I\chi$ for π^0 and $\bar{\chi}\gamma_5\chi$ for π^+ and the right plot for local operators $\bar{\chi}\gamma_4\gamma_5\tau_3\chi$ for π^0 and $\bar{\chi}\gamma_4\chi$ for π^+ . The curves give the ratio arising from the mass difference determined by the full fit.

this determination to estimate the consequences of smaller lattice spacing, for instance less than 20% pion splitting for $m(\pi^+) = 200$ MeV provided $r_0/a > 8.2$.

4. Flavour singlet PS meson: η_2

In QCD, the flavour singlet pseudoscalar meson acquires a mass through the anomaly, so is not a goldstone boson. It is important to check that this feature, which is linked to topological charge fluctuations, is reproduced in lattice evaluations. With $N_f = 2$ degenerate quarks, the flavour singlet pseudoscalar meson (called η_2) is related to the experimental $\eta'(958)$ and is expected [9] to have a mass around 800 MeV. We fit the η_2 correlators (2×2 matrix with local and non-local operator $\bar{\psi}\gamma_5\psi \rightarrow \bar{\chi}\tau_3\chi$ at maximal twist) for t -range 3-10 with 2 states. We compare results from TMQCD [3] with older results (see ref. [10] for a review) in fig. 3. Note that the ETMC results are at substantially smaller quark masses. We see that the η_2 mass is consistent with a constant behaviour in the chiral limit with $m(\eta_2) \approx .88$ GeV ($r_0 m(\eta_2) = 2$).

We now discuss why the errors are so large for the η_2 , despite the fact that we measure all t and t' , we use many gauge configurations and stochastic errors are small. The origin of the problem is that the signal for the disconnected part of the correlator comes from only a small part of the total data sample. For instance (at $\mu = 0.004$ with 48 t -values for 888 gauge configurations) with $|t' - t| = 10$, 2.1% of the data contributes 26% of the signal. Thus the statistical impact of the data set is smaller than expected since parts of the data have big fluctuations (in a fermionic loop related to topological charge density). So even more configurations are needed to get reliable and small errors in the case of disconnected contributions.

**Figure 2:** Pion charge splitting

5. Vector mesons

We compare the local-local correlators (including disconnected parts for the neutral meson) for vector mesons in fig. 4. Note that the disconnected contribution is negligible for the vector coupling to neutral ρ -mesons. We find that the ratio is consistent with constant (ie no mass splitting) and the value of that constant can be related to renormalisation constants. For the vector coupling to neutral ρ -mesons this implies $(Z_A/Z_V)^2 \approx 1.5$ at the finer lattice spacing, consistent with [12]. We find agreement with a ratio of 1.0 for the tensor couplings, as expected since there is only one tensor renormalisation which then cancels.

From fits to a 2×2 matrix of correlators (from the connected neutral contribution only here in t -range 8-18) we obtain ρ^0 masses (in lattice units). We also report values for the ρ^+ masses [3] (from fits to a 4×4 matrix of correlators). These values are consistent with no flavour splitting for vector mesons as expected [11].

β	3.9	3.9	3.9	4.05	4.05
μ	0.004	0.004	0.0085	0.003	0.006
L	32	24	24	32	32
$am(\rho^0)$.400(25)	.395(17)	.419(17)	.372(29)	.346(12)
$am(\rho^+)$.416(12)	.404(22)	.428(8)	.337(20)	.337(12)

We now consider decay transitions from the vector meson to two pions following the methods used in ref. [13]. The transition $\rho^0(0) \rightarrow \pi^+(1)\pi^-(1)$, where $q(\pi) = \pm 2\pi/L$, even for the lightest pion ($M(\pi^+) \approx 300$ MeV, $L = 24a$, $a = 0.086$ fm) is not open (note, however, that decay for a ρ^0 meson with non-zero momentum is open). For $\rho^0(0)$, there is an energy splitting of $\Delta ma = 0.19$ assuming that the two pion state has twice the energy of a π^+ with appropriate momentum.

We show the normalised result in fig. 5 where it is compared to a two-state model [13]. The

value of the transition amplitude x is consistent with the empirical ρ decay width. One can also estimate the effect of this mixing transition on the ρ mass using the two-state model. This gives a downward shift of ma of .02 (eg from .41 to .39 for the ρ mass). This shift, induced by the proximity of the lightest two pion level, is comparable to our statistical error in determining the ρ mass. This suggests that we do not yet see major modifications of our ρ meson masses from mixing with the decay channel.

6. Flavour singlet scalar mesons

There is considerable confusion in allocating the experimental flavour-singlet scalar meson (f_0) spectrum to specific content: since scalar glueball, $\bar{u}u + \bar{d}d$, $\bar{s}s$, $\pi\pi$ and/or KK in an S-wave, etc., can all contribute. Lattice QCD can help considerably here, but it will be difficult as we now illustrate.

We took a first look in TMQCD (here $M(\pi^+) \approx 300$ MeV, $a = 0.086$ fm, $La = 2.1$ fm) and made a 2 state fit (t -range 6 to 23) to the 6×6 correlator matrix (P , S , A_4 both local and fuzzed at sink and source, including disconnected contributions). We find states $ma = .103(5)$ (π^0) and $m'a = .227(28)$ (f_0 , energy consistent with $2m(\pi^0)$).

Thus in the scalar channel we find a clear signal - but at the mass of two pions. This is not unexpected - but emphasises the problems of studying scalar mesons with light quarks in dynamical lattice gauge theory, where the light two-body state will dominate the correlators.

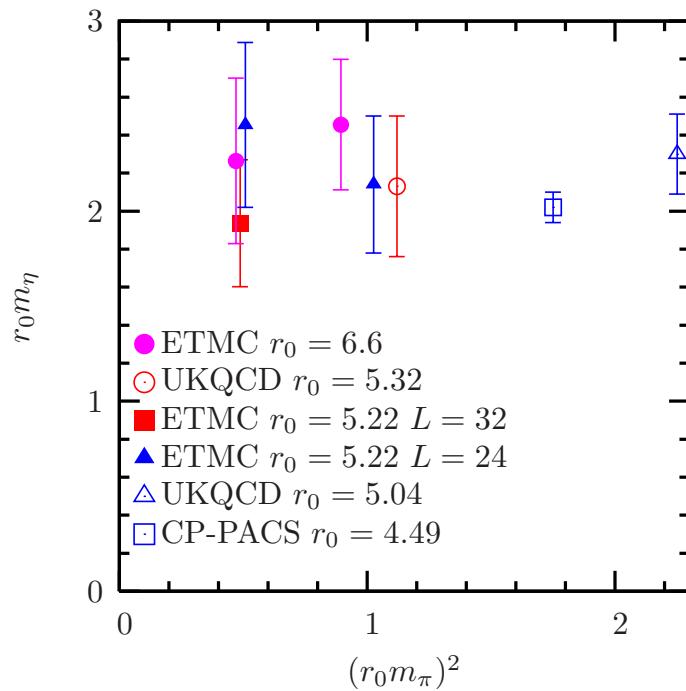


Figure 3: η_2 mass versus quark mass. Here we summarise results with light pions and $a < 0.1$ fm ($r_0/a > 4.5$).

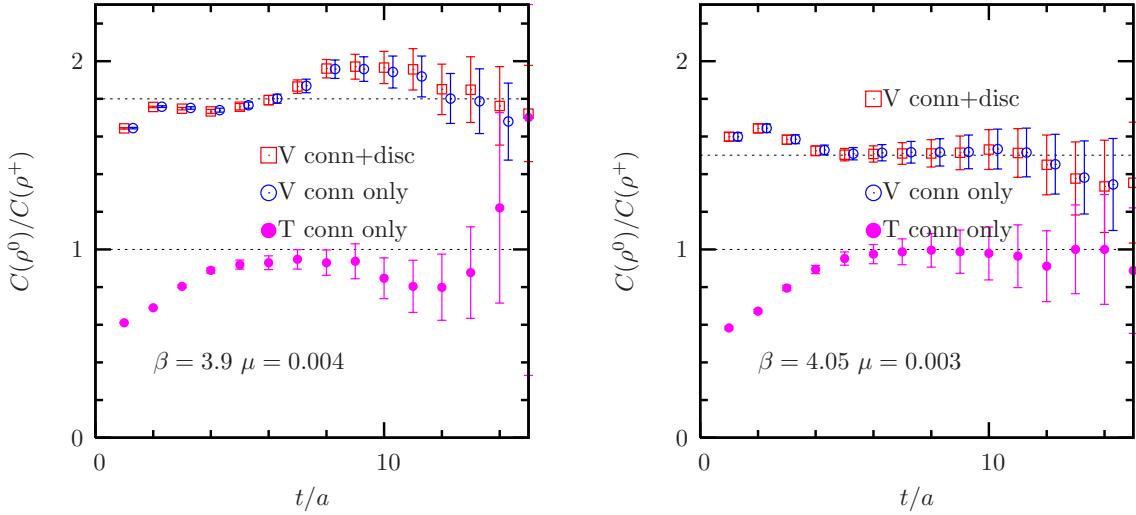


Figure 4: Ratios of correlator for neutral ρ -mesons to charged. The dotted lines guide the eye in the case that there is no mass splitting.

7. Summary

TMQCD allows efficient evaluation of disconnected contributions using a powerful variance reduction method.

The π charge splitting goes to zero like a^2 as expected, and the sign is consistent with the nature of the phase transition.

The flavour singlet pseudoscalar meson (η_2) has been studied to lighter quarks than previously and is consistent with a mass of around 800 MeV in chiral limit.

For vector mesons we find negligible charge splitting and the decay (transition to $\pi \pi$) is accessible to study.

Our study of the flavour singlet scalar meson (f_0) with light dynamical quarks, finds the expected $\pi \pi$ contribution which will obscure further study of heavier states.

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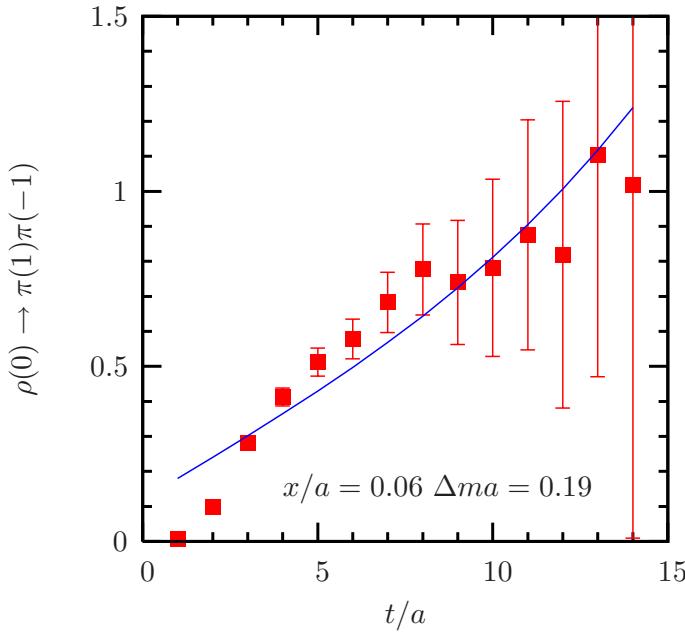


Figure 5: Transition $\rho^0 \rightarrow \pi^+ \pi^-$ from the lattice (with $M(\pi^+) \approx 300$ MeV, $a = 0.086$ fm, $La = 2.1$ fm). The solid line is from a two state model (ρ at rest and two pions with momentum $q = \pm 2\pi/L$) with energy gap $\Delta ma = 2aE(\pi) - am(\rho) = 0.19$ and transition amplitude x/a .

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